

Recruiting, Retaining and Rewarding Top Tier Teachers



# Math Principles

## Geometry

# Geometry

- ▶ How do you **FIND THE DISTANCE BETWEEN TWO POINTS?**
- ▶ To find the distance between points, use the distance formula.  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- ▶ If you forget it remember that you can draw a right triangle where the differences between the X's is one leg and the difference between the Y's is the other. When you draw this triangle you can find distances or points whichever the question asks for

# Geometry

- ▶ How do you **USE TWO POINTS TO FIND THE SLOPE?**
- ▶ In mathematics, the slope of a line is often called  $m$ .
- ▶ Slope =  $m = (y - y) / (x - x)$
- ▶ For example to find the slope of the line that contains the points  $(5, 7)$  and  $(3, -1)$
- ▶  $(7 - -1) / (5 - 3) = 8 / 2 = 4$

# Geometry

- ▶ How do you **FIND THE SLOPE IF GIVEN AN EQUATION?**
- ▶ To find the slope of a line from an equation, put the equation into the slope-intercept form:
- ▶  $y = mx + b$
- ▶ The slope is  $m$  and the  $y$  intercept is  $b$
- ▶ For example to find the slope of the equation
$$15x + 3y = 21$$
$$3y = -15x + 21$$
$$y = -5x + 7$$

# Geometry

- ▶ How do you **USE AN EQUATION TO FIND THE X OR Y INTERCEPT?**
- ▶ To find the slope of a line from an equation, put the equation into the slope-intercept form:
- ▶  $y = mx + b$
- ▶ If you are asked for the y intercept the answer is b or just plug in  $x = 0$ . To find the x intercept plug in  $y = 0$  and solve.
- ▶ For example if you are given the equation  $y = 5x + 10$
- ▶ The y intercept is 10 the x intercept is found as follows
- ▶  $0 = 5x + 10; \quad \rightarrow \quad -10 = 5x; \quad \rightarrow \quad -2 = x$

# Geometry

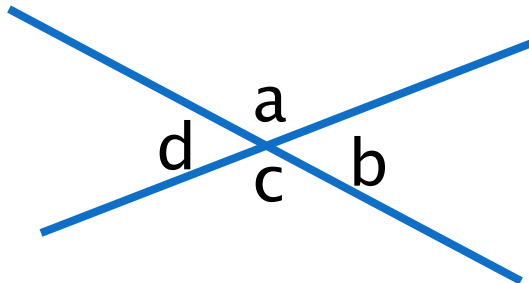
- ▶ What is the **EQUATION FOR A CIRCLE** graph?
- ▶ The equation for a circle of radius  $r$  and centered at  $(h, k)$  is
- ▶  $(x - h)^2 + (y - k)^2 = r^2$
- ▶ For example if you were given the following equation  $(x - 3)^2 + (y - 5)^2 = 25$  then it would be for a circle with a center at  $(3, 5)$  and radius of 5. You could be asked to go from the equation to the graph or vice versa.
- ▶ \*This is always tested on the ACT but rarely if ever on SAT

# Geometry

- ▶ How do you **EQUATION FOR A PARABOLA**
- ▶ The graph of an equation in the form  $y = ax^2 + bx + c$  is a parabola.
- ▶ Plug into your calculator and see what happens.

# Geometry

- ▶ What are the properties of **INTERSECTING LINES**?
- ▶ Give two intersecting lines, the adjacent angles are supplementary (add to  $180^\circ$ ) and vertical angles are equal.

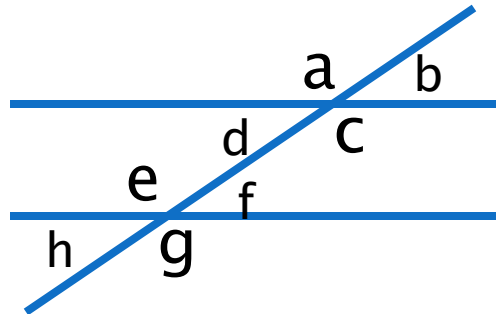


- ▶ For Example: In the figure above, the angles marked  $a^\circ$  and  $b^\circ$  are adjacent and supplementary, so  $a + b = 180$ .
- ▶ If you were told that  $a = 110$  then  $c = 110$  and  $b = 70$



# Geometry

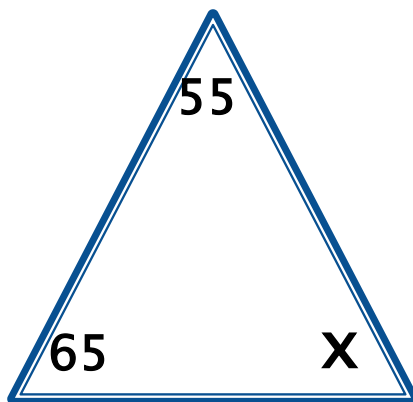
- ▶ How do you **PARALLEL LINES AND TRANSVERSALS**
- ▶ A transversal across parallel lines forms four equal acute angles and four equal obtuse angles. Use the same rules as intersecting lines as well.



- ▶ Here, line 1 is parallel to line 2. Angles a, c, e, and g are obtuse, so they are all equal. Angles b, d, f, and h are acute, so they are all equal. Also any of the acute angles is supplementary to any of the obtuse angles.

# Geometry

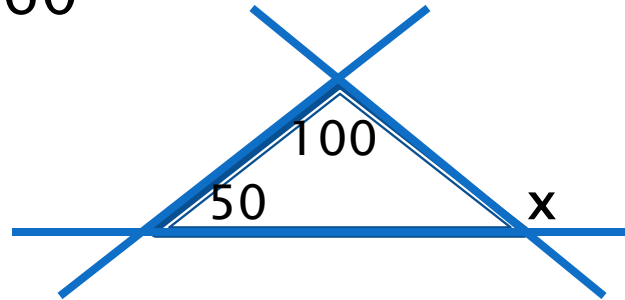
- ▶ How do you find **INTERIOR ANGLES OF A TRIANGLE**?
- ▶ The three angles of any triangle add up to  $180^\circ$ .



- ▶ For Example In the figure above,  $x + 55 + 65 = 180$ , so  $x = 60$ .

# Geometry

- ▶ How do you find the **EXTERIOR ANGLES OF A TRIANGLE?**
- ▶ An exterior angle of a triangle is equal to the sum of the remote interior angles.
- ▶ All three exterior angles of a triangle add up to  $360^\circ$



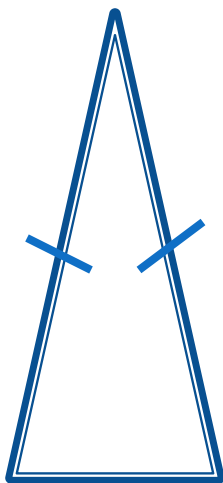
- ▶ In the figure above, the exterior angle labeled  $x^\circ$  is equal to the sum of the remote interior angles:
- ▶  $x = 50 + 100 = 150$ .

# Geometry

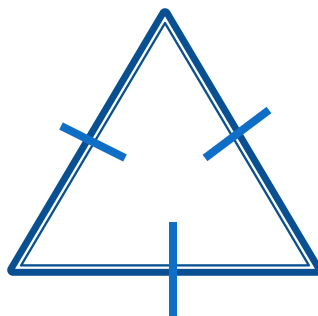
- ▶ How do you find **THE THIRD SIDE OF A GENERAL TRIANGLE?**
- ▶ The triangle inequality states that the third side of any triangle will be smaller than the sum of the other two or bigger than the difference between them
- ▶ For example if a triangle has sides of 3 and 7 the third side is less than 10 and greater than 4.

# Geometry

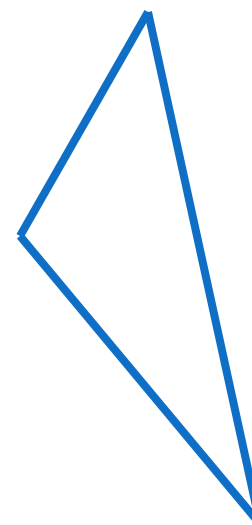
- ▶ What are the types of triangles?



Isosceles  
2 equal sides



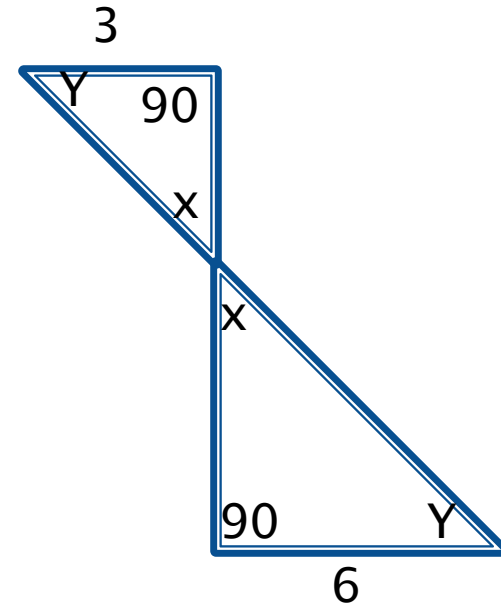
Equilateral  
3 equal sides



Scalene  
no sides equal

# Geometry

- ▶ How do you solve problems with **SIMILAR TRIANGLES**?
- ▶ Similar triangles have the same shape: corresponding angles are equal and corresponding sides are proportional.

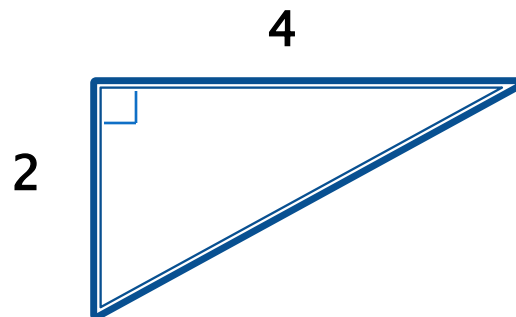


The figure above is common for similar triangles. Corresponding sides are proportional

# Geometry

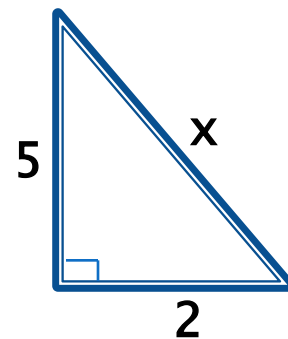
- ▶ How do you find the **AREA OF A TRIANGLE?**
- ▶ Area of Triangle =  $(\frac{1}{2})(\text{base})(\text{height})$
- ▶ Remember that the base doesn't have to be on the bottom. The height is perpendicular to the chosen base.

- ▶ For example in the triangle to the right the area is  
 $A = (\frac{1}{2})(2)(4) = 4$



# Geometry

- ▶ How do you **FIND THE LENGTH OF AN UNKNOWN SIDE OF A RIGHT TRIANGLE?**
- ▶ For all **right triangles** use the pythagorean theorem.
- ▶  $(\text{leg1})^2 + (\text{leg2})^2 = (\text{hypotenuse})^2$
- ▶ For example for the triangle to the right  
the right  
$$x^2 = 5^2 + 2^2 = \sqrt{29}$$





# Geometry

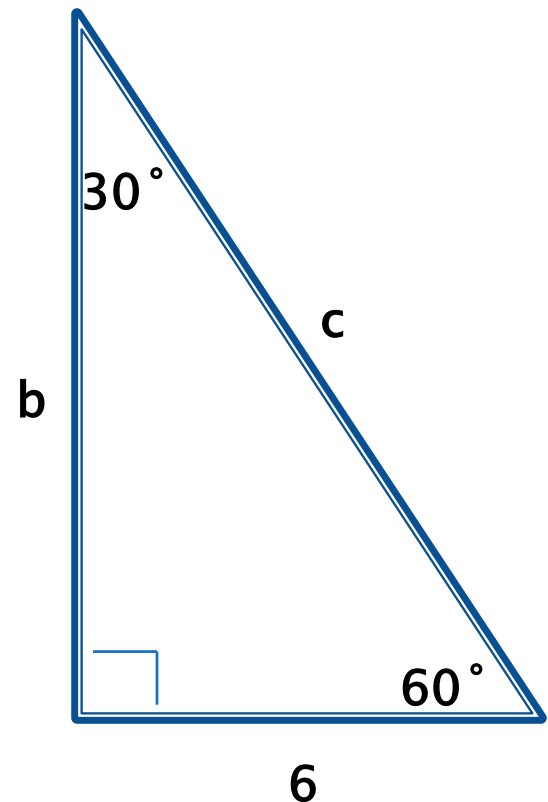
- ▶ How do you work with ***3-4-5*** RIGHT TRIANGLES
- ▶ If the ratio of two legs of a right triangle is 3:4 or if a leg to the hypotenuse is 3:5 or 4:5 then it is a 3-4-5 triangle. This is a shortcut to the pythagorean theorem.
- ▶ For example if two sides of a right triangle are 12 and 16 then what is the hypotenuse?  
 $3 \times 4 = 12$  and  $4 \times 4 = 16$  so  $5 \times 4 = 20$   
The hypotenuse is 20.

# Geometry

- ▶ How do you work with ***5-12-13*** RIGHT TRIANGLES
- ▶ If the ratio of two legs of a right triangle is 5:12 or if a leg to the hypotenuse is 5:13 or 12:13 then it is a 5-12-13 triangle. This is a shortcut to the pythagorean theorem.
- ▶ For example if two sides of a right triangle are 10 and 24 then what is the hypotenuse?  
 $5 \times 2 = 10$  and  $12 \times 2 = 24$  so  $13 \times 2 = 26$   
The hypotenuse is 26.

# Geometry

- ▶ How do you find the sides in a 30–60–90 triangle?
- ▶ The sides across from the  $30^\circ$ ,  $60^\circ$  &  $90^\circ$  angles are  $x$ ,  $x\sqrt{3}$  and  $2x$  respectively
- ▶ For example in the triangle to the right, what are the values of  $b$  and  $c$ ?  
 $c = 12$  and  $b = 6\sqrt{3}$



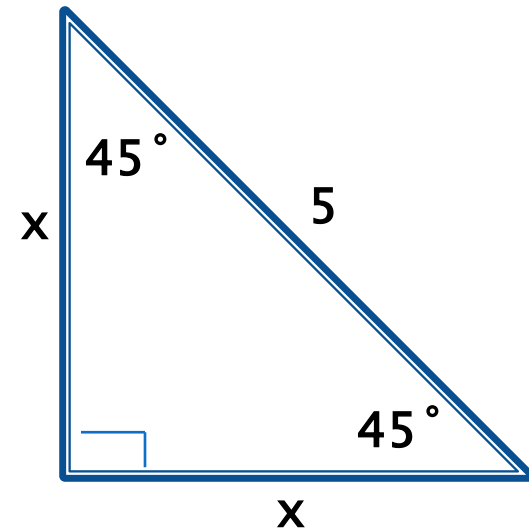
# Geometry

- ▶ How do you find the sides in a ***45-45-90 Triangle***?
- ▶ The sides across from the  $45^\circ$ ,  $45^\circ$  &  $90^\circ$  are  $x$ ,  $x$  and  $x\sqrt{2}$  respectively.
- ▶ For example in the triangle to the right find the length of one of the legs.

$$5 = x\sqrt{2}$$

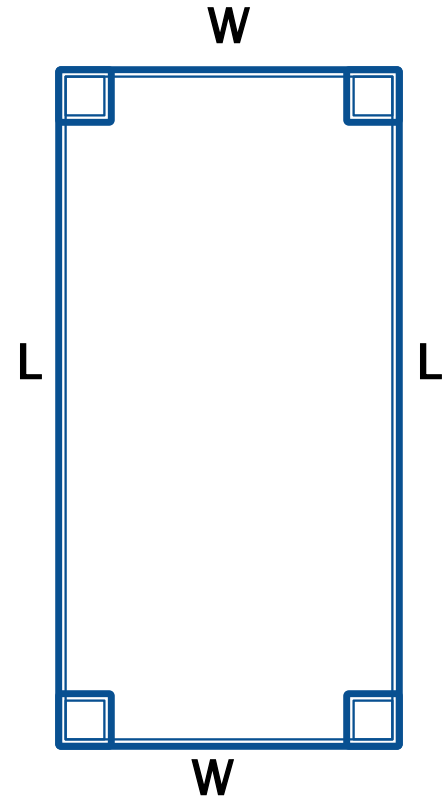
$$5/\sqrt{2} = x$$

$$(5/2)\sqrt{2}$$



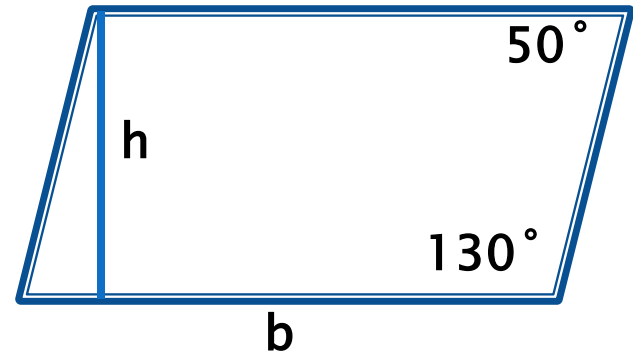
# Geometry

- ▶ How do you work with ***RECTANGLES?***
- ▶ A rectangle is a four-sided figure with four right angles. Opposite sides are equal. Diagonals are equal.
- ▶ The perimeter is  $P = 2(\text{length} + \text{width})$ .
- ▶ The area is  $A = (\text{length})(\text{width})$



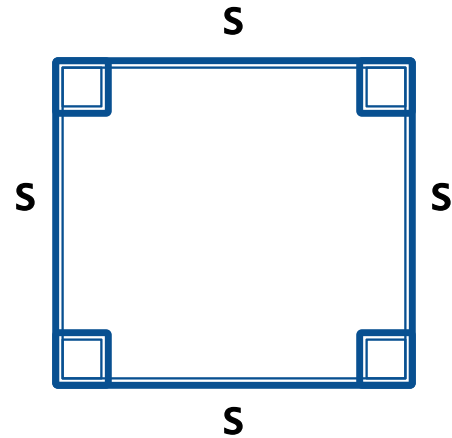
# Geometry

- ▶ How do you work with ***PARALLELOGRAMS?***
- ▶ A parallelogram has two pairs of parallel sides. Opposite sides are equal. Opposite angles are equal. Consecutive angles add up to  $180^\circ$ .
- ▶ Area = (base)(height)



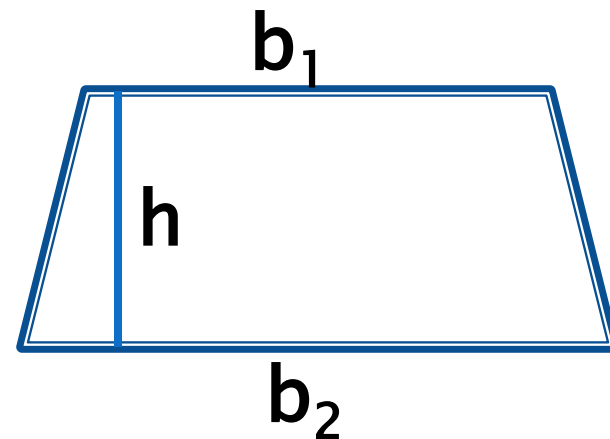
# Geometry

- ▶ How do you work with **SQUARES?**
- ▶ A square is a rectangle with 4 equal sides.
- ▶ The perimeter of a square is  $P = 4s$
- ▶ The area is  $A = s^2$
- ▶ Diagonals =  $s\sqrt{2}$



# Geometry

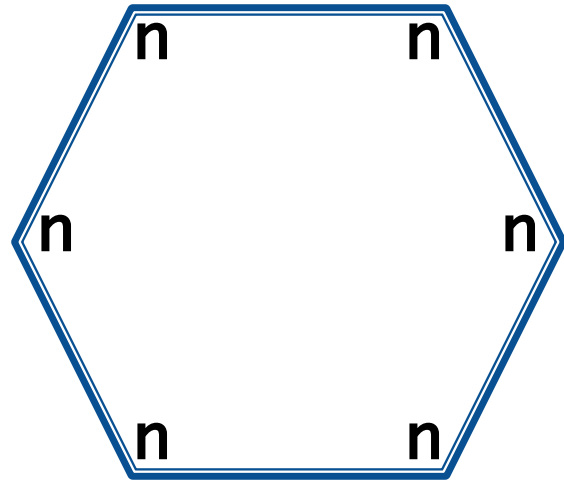
- ▶ How do you work with **TRAPEZOIDS?**
- ▶ A trapezoid is a quadrilateral with one pair of parallel sides and one pair of nonparallel sides.
- ▶ Area =  $\frac{1}{2} (b_1 + b_2)h$





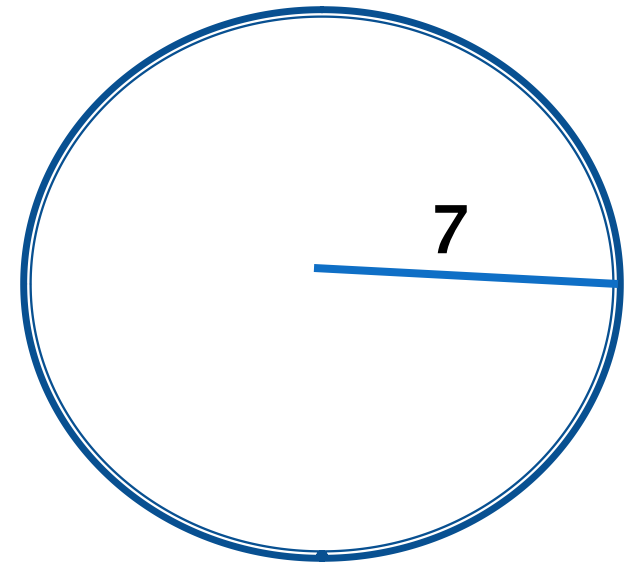
# Geometry

- ▶ How do you find the **INTERIOR ANGLES OF A POLYGON?**
- ▶ The sum of the measures of the interior angles of a polygon is  $(n - 2)180$ , where  $n$  is the number of sides.
- ▶ If you are told it is a regular polygon then that means that all of the angles are equal to each other.
- ▶ For example in the hexagon the total interior sum is  $(6-2)180 = 720$
- ▶ If it is a regular hexagon then each angle is  $(720/6) = 120$



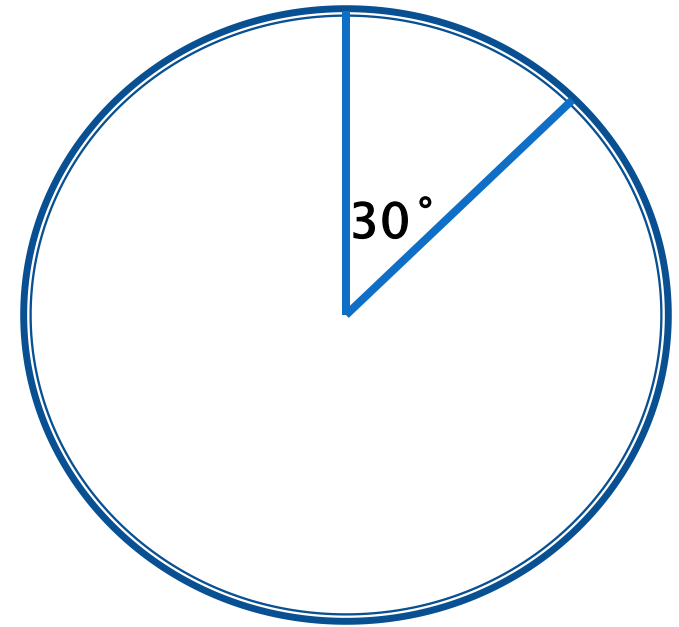
# Geometry

- ▶ How do you find the **CIRCUMFERENCE OF A CIRCLE?**
- ▶  $C = 2\pi r$  or  $C = \pi d$
- ▶ For example in the circle to the right  
 $C = 2\pi 7 = 14\pi$



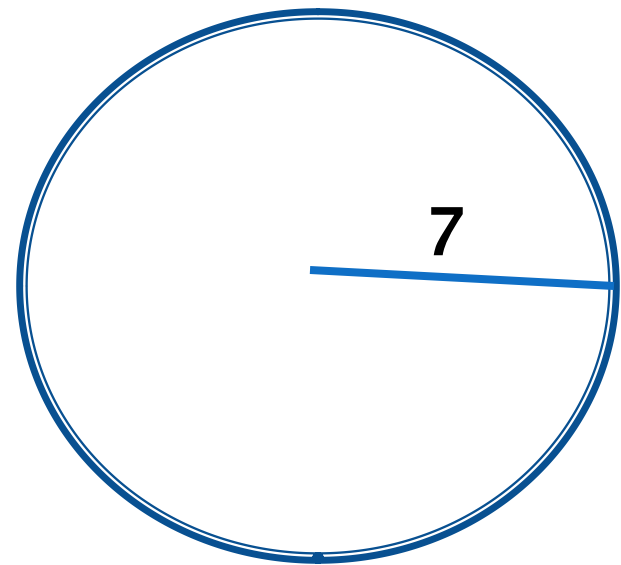
# Geometry

- ▶ How do you find the **LENGTH OF AN ARC**?
- ▶ An arc is part of the circumference. If  $n$  is the measure of the arc's central angle, then the formula is:
- ▶ Arc Length =  $(n/360)(2\pi r)$
- ▶ For example in the figure the radius is 12 and the measure of the central angle is  $30^\circ$ . The arc length is
- ▶ Arc =  $(30/360)(2\pi 12) = 2\pi$



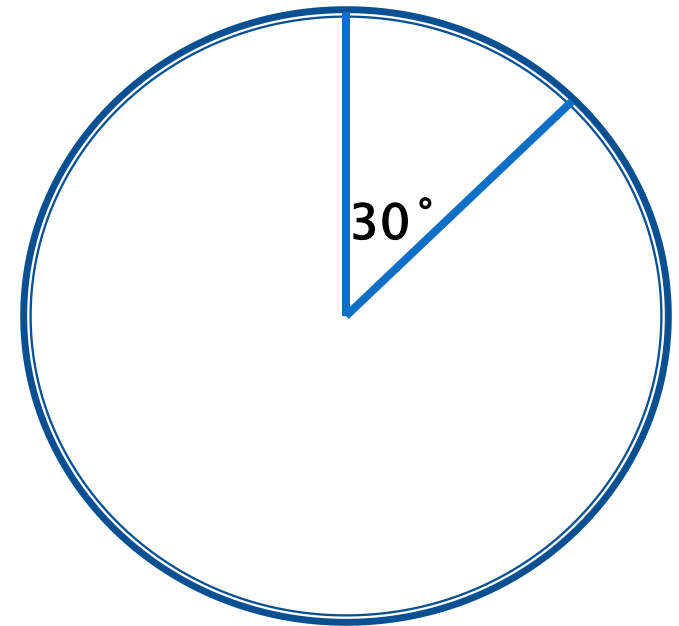
# Geometry

- ▶ How do you find the **AREA OF A CIRCLE?**
- ▶ Area of a circle =  $\pi r^2$
- ▶ For example the area of the circle to the right is  $A = \pi(7)^2 = 49\pi$ .



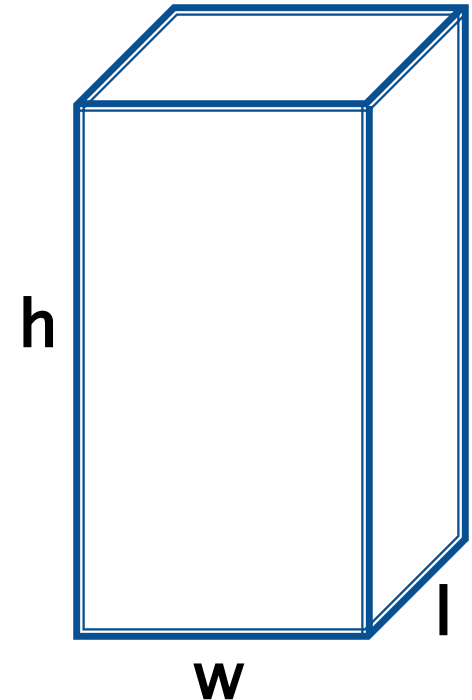
# Geometry

- ▶ How do you find the **AREA OF A SECTOR**?
- ▶ A sector is part of a circle's area. If  $n$  is the measure of the sector's central angle, then the formula is:
- ▶ Area of a Sector =  $(n/360)(\pi r^2)$
- ▶ In the figure above, the radius is 6 and the measure of the sector's central angle is  $30^\circ$ .  
 $A = (30/360)(\pi 6^2) = 3\pi$



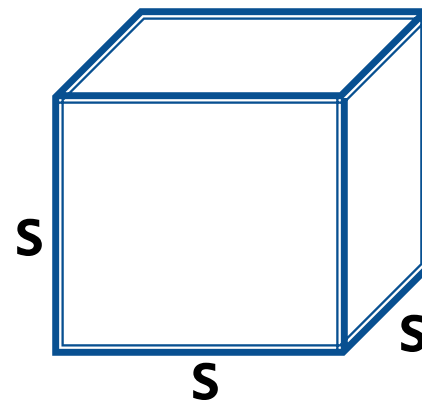
# Geometry

- ▶ How do you find the **SURFACE AREA OF A RECTANGULAR SOLID**?
- ▶ The surface of a rectangular solid consists of 3 pairs of identical faces. To find the surface area, find the area of each face and add them up. If the length is  $l$ , the width is  $w$ , and the height is  $h$ , the formula is:
- ▶ Surface Area =  $2lw + 2wh + 2lh$



# Geometry

- ▶ How do you find the **VOLUME OF A RECTANGULAR SOLID?**
- ▶ Volume of a Rectangular Solid =  $lwh$
- ▶ If it is a cube then all sides are equal so  
 $V = s^3$



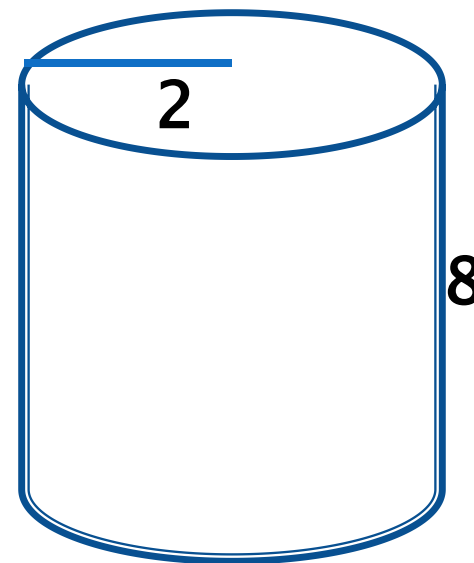
# Geometry

- ▶ How do you find the **VOLUME OF A CYLINDER?**

- ▶  $V = \pi r^2 h$

- ▶ For example the volume of a cylinder to the right is

$$V = \pi(2^2)(8) = 32\pi$$





# Geometry

- ▶ How do you find the **VOLUME OF A CONE?**
- ▶  $V = (1/3) \pi r^2 h$
- ▶ Never tested on SAT but may be on ACT

# Geometry

- ▶ How do you find the **VOLUME OF A SPHERE?**
- ▶  $V = (4/3)\pi r^3$
- ▶ Never tested on SAT but may be on ACT