

Recruiting, Retaining and Rewarding Top Tier Teachers



Math Principles

Basic Math

- ▶ What is an **INTEGER/NONINTEGER**?
- ▶ Integers are whole numbers; they include negative whole numbers and zero.
- ▶ For example -7, 0, 18 are integers, 1.5 is not.

Basic Math

- ▶ What is a **REAL/IMAGINARY** number?
- ▶ A real number is a number that has a location on the number line.
- ▶ Imaginary numbers are numbers that involve the square root of a negative number.
- ▶ For example $\sqrt{-4}$ is an imaginary number.

Basic Math

- ▶ What is an **UNDEFINED** number?
- ▶ Generally indicates division by zero.

Basic Math

- ▶ What is a **RATIONAL OR IRRATIONAL NUMBER?**
- ▶ A rational number is a number that can be expressed as a ratio of two integers.
- ▶ Irrational numbers are real numbers—they have locations on the number line—they just can't be expressed precisely as a fraction or decimal.
- ▶ For example the most commonly tested irrational numbers are $\sqrt{2}$, $\sqrt{3}$, and π .

Basic Math

- ▶ How do you do problems with **ADDING/SUBTRACTING SIGNED NUMBERS?**
- ▶ To add a positive and a negative, first ignore the signs and find the positive difference between the number parts. Then attach the sign of the original number with the larger number part.
- ▶ For example, to add 23 and -34, first we ignore the minus sign and find the positive difference between 23 and 34—that's 11. Then we attach the sign of the number with the larger number part—in this case it's the minus sign from the -34. So, $23 + (-34) = -11$.
- ▶ Make subtraction situations simpler by turning them into addition.
- ▶ For example, think of $-17 - (-21)$ as $-17 + (+21)$.
- ▶ To add or subtract a string of positives and negatives, first turn everything into addition. Then combine the positives and negatives so that the string is reduced to the sum of a single positive number and a single negative number.

Basic Math

- ▶ How do you do problems with **MULTIPLYING/DIVIDING SIGNED NUMBERS?**
- ▶ To multiply and/or divide positives and negatives, treat the number parts as usual and attach a negative sign if there were originally an odd number of negatives.
- ▶ To multiply -4 , -5 , and -7 , first multiply the number parts: $4 \times 5 \times 7 = 140$. Then go back and note that there were three—an odd number negatives, so the product is negative: $(-4) (-5) (-7) = -140$.

Basic Math

- ▶ What is the rule for order of operations, **PEMDAS**?
- ▶ When performing multiple operations, remember **PEMDAS**, which means Parentheses first, then Exponents, then Multiplication and Division (left to right), then Addition and Subtraction (left to right).
- ▶ In the expression $10 - 2(6 - 2)^2 + 8/4$
- ▶ Parentheses first: $(6 - 2) = 4$. Then the exponent: $4^2 = 16$. Then multiplication and division $10 - (2 \times 16) + 8/2$ so $10 - 32 + 4 = -18$

Basic Math

- ▶ What do you do with **ABSOLUTE VALUE**?
- ▶ Treat absolute value signs a lot like parentheses.
- ▶ Do what's inside them first and then take the absolute value of the result. Don't take the absolute value of each piece between the bars before calculating.
- ▶ For example In order to calculate $|-5 + 3|$ do not do $5 + 3 = 8$, instead perform the operation in the absolute value $|-5 + 3| = |-2|$, then take the positive sign of the number $|-2| = 2$
- ▶ So overall $|-5 + 3| = |-2| = 2$

Basic Math

- ▶ How do you COUNT CONSECUTIVE INTEGERS?
- ▶ To count consecutive integers, subtract the smallest from the largest and add 1.
- ▶ For example to count the integers from 13 through 131, subtract: $131 - 13 = 118$. Then add 1: $118 + 1 = 119$.

Basic Math

- ▶ **What is a FACTOR/MULTIPLE?**
- ▶ The factors of integer n are the positive integers that divide into n with no remainder. The multiples of n are the integers that n divides into with no remainder.
- ▶ For example 3 is a factor of 6 while 12 and 24 are multiples of 6. 6 is both a factor and a multiple of itself.

Basic Math

- ▶ What is meant by **PRIME FACTORIZATION**?
- ▶ A prime number is a positive integer that has exactly two positive integer factors: 1 and the integer itself. The first eight prime numbers are 2, 3, 5, 7, 11, 13, 17, and 19.
- ▶ Remember that 1 is not prime but 2 is
- ▶ To find the prime factorization of an integer, just keep breaking it up into factors until all the factors are prime. To find the prime factorization of 90, for example, you could begin by breaking it into 9×10 : then further into $3 \times 3 \times 5 \times 2$

Basic Math

- ▶ What are **RELATIVE PRIMES**?
- ▶ To determine whether two integers are relative primes, break them both down to their prime factorizations.
- ▶ For example: $21 = 3 \cdot 7$, and $40 = 5 \cdot 2 \cdot 2 \cdot 2$. They have no prime factors in common, so 21 and 40 are relative primes.

Basic Math

- ▶ What is a **COMMON MULTIPLE**?
- ▶ You can always get a common multiple of two numbers by multiplying them, but, unless the two numbers are relative primes, the product will not be the least common multiple.
- ▶ For example, to find a common multiple for 11 and 10, you could just multiply: $11 \times 10 = 110$.

Basic Math

- ▶ How do you find the **LEAST COMMON MULTIPLE (LCM)**?
- ▶ To find the least common multiple, check out the multiples of the larger number until you find one that's also a multiple of the smaller.
- ▶ To find the LCM of 12 and 15, begin by taking the multiples of 15: 15 is not divisible by 12; 30's not; nor is 45. But the next multiple of 15, 60, is divisible by 12, so it's the LCM.

Basic Math

- ▶ What is a **GREATEST COMMON FACTOR (GCF)**?
- ▶ To find the greatest common factor, break down both numbers into their prime factorizations and take all the prime factors they have in common.
- ▶ $36 = 2 \cdot 2 \cdot 3 \cdot 3$, and $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$. What they have in common is two 2s and one 3, so the GCF is $= 2 \cdot 2 \cdot 3 = 12$

Basic Math

- ▶ How do you perform **Operations with EVEN/ODD numbers?**
- ▶ To predict whether a sum, difference, or product will be even or odd, just take simple numbers like 1 and 2 and see what happens.
- ▶ There are rules—“odd times even is even,” for example—but there’s no need to memorize them. What happens with one set of numbers generally happens with all similar sets

Basic Math

- ▶ How do you Find MULTIPLES OF 2 AND 4?
- ▶ An integer is divisible by 2 if the last digit is even.
- ▶ An integer is divisible by 4 if the last two digits form a multiple of 4.
- ▶ The last digit of 456 is 6, which is even, so 456 is a multiple of 2. The last two digits make 56, which is divisible by 4, so 456 is a multiple of 4.

Basic Math

- ▶ How do you Find **MULTIPLES OF 3 AND 9**?
- ▶ An integer is divisible by 3 if the sum of its digits is divisible by 3.
- ▶ An integer is divisible by 9 if the sum of its digits is divisible by 9.
- ▶ The sum of the digits in 873 is 18, which is divisible by 3 and 9, so 873 is divisible by both.

Basic Math

- ▶ How do you find **MULTIPLES OF 5 AND 10?**
- ▶ An integer is divisible by 5 if the last digit is 5 or 0.
- ▶ An integer is divisible by 10 if the last digit is 0.
- ▶ The last digit of 665 is 5, so 665 is a multiple 5 but not a multiple of 10.

Basic Math

- ▶ How do you solve questions with **REMAINDERS?**
- ▶ The remainder is the whole number left over after division. It is always less than the divisor in the equation.
- ▶ 487 is 2 more than 485, which is a multiple of 5, so when 487 is divided by 5, the remainder will be 2.
- ▶ **Remember that remainders will always be less than your divisor.**

Basic Math

- ▶ How do you **REDUCE FRACTIONS?**
- ▶ To reduce a fraction to lowest terms, factor out and cancel all factors the numerator and denominator have in common.

$$\text{▶ } \frac{115}{25} = \frac{(5)(23)}{(5)(5)} = \frac{23}{5}$$

Basic Math

- ▶ How do you **ADD/SUBTRACT FRACTIONS?**

- ▶ To add or subtract fractions, first find a common denominator, and then add or subtract the numerators.

- ▶
$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}$$

Basic Math

- ▶ How do you **MULTIPLY FRACTIONS?**
- ▶ To multiply fractions, multiply the numerators and multiply the denominators.

$$\text{▶ } \frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8}$$

Basic Math

- ▶ How do you **DIVIDE FRACTIONS?**
- ▶ To divide fractions, invert the second one and multiply.

$$\text{▶ } \frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$$

Basic Math

- ▶ How do you **CONVERT A MIXED NUMBER TO AN IMPROPER FRACTION?**
- ▶ To convert a mixed number to an improper fraction, multiply the whole number part by the denominator, then add the numerator. The result is the new numerator (over the same denominator).
- ▶ To convert $7 \frac{1}{3}$ first multiply 7 by 3, then add 1, to get the new numerator of 22. Put that over the same denominator, 3, to get $\frac{22}{3}$

Basic Math

- ▶ How do you **CONVERT AN IMPROPER FRACTION TO A MIXED NUMBER**
- ▶ To convert an improper fraction to a mixed number, divide the denominator into the numerator to get a whole number quotient with a remainder. The quotient becomes the whole number part of the mixed number, and the remainder becomes the new numerator—with the same denominator.
- ▶ For example, to convert $108/5$, first divide 5 into 108, which yields 21 with a remainder of 3 so $21 \frac{3}{5}$

Basic Math

- ▶ What is a **RECIPROCAL**?
- ▶ To find the reciprocal of a fraction, **switch the numerator and the denominator.**
- ▶ The reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$. The reciprocal of 5 is $\frac{1}{5}$
- ▶ The product of reciprocals is 1.

Basic Math

- ▶ How do you **COMPARE FRACTIONS?**
- ▶ One way to compare fractions is to express them with a common denominator.
- ▶ Another way to compare fractions is to convert them both to decimals.
- ▶ So $\frac{3}{4}$ vs. $\frac{5}{7}$ is $\frac{21}{28}$ vs $\frac{20}{28}$.
- ▶ Or .75 vs. .714

Basic Math

- ▶ How do you **CONVERT FRACTIONS TO DECIMALS?**
- ▶ To convert a fraction to a decimal, divide the bottom into the top.
- ▶ To convert $5/8$ divide 8 into 5, yielding .625.

Basic Math

- ▶ What do you do with a **REPEATING DECIMAL**?
- ▶ To find a particular digit in a repeating decimal, note the number of digits in the cluster that repeats. If there are 2 digits in that cluster, then every 2nd digit is the same. If there are 3 digits in that cluster, then every 3rd digit is the same. And so on.
- ▶ For example, the decimal equivalent of $1/27$ is $.037037037\dots$, which is best written $.0\overline{37}$.
- ▶ There are 3 digits in the repeating cluster, so every 3rd digit is the same: 7. To find the 50th digit, look for the multiple of 3 just less than 50—that's 48. The 48th digit is 7, and with the 49th digit the pattern repeats with 0. The 50th digit is 3.

Basic Math

- ▶ How do you **IDENTIFY THE PARTS AND THE WHOLE?**
- ▶ The key to solving most story problems involving fractions and percents is to identify the part and the whole. Usually you'll find the part associated with the verb is/are and the whole associated with the word of.
- ▶ In the sentence, "Half of the boys are blonds," the whole is the boys ("of the boys"), and the part is the blonds ("are blonds").